Evolution of Space Charge Dominated Beam Envelope in a Compact Cyclotron

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Abstract. This paper presents the results of studies on the behavior of transverse envelopes of space charge dominated beam along the accelerated orbits in a compact cyclotron. We have used coupled beam envelope equations with acceleration and assumed the beam as a uniform ellipsoidal bunch. A detailed investigation on the amplitude growth and oscillations in the beam envelopes has been carried out by changing the initial beam parameters. We have obtained the proper matching conditions by optimizing the input parameters and also estimated the maximum beam current that can be transported through a given aperture of the cyclotron focusing channel.

Keywords: Cyclotron, Space charge dominated beams, Beam optics.
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INTRODUCTION

At the Variable Energy Cyclotron Centre, we are developing a 10 MeV, 5 mA compact proton cyclotron [1]. Proton beam (20 mA, 80 keV) from a 2.45 GHz microwave ion source will be bunched and injected axially in the central region of the cyclotron by a spiral inflector. It is well known that space charge effect alters the beam dynamics in a cyclotron and sets a limit on the beam current that can be accelerated. For optimum performance, the input phase ellipses of the injected beam must be matched to the acceptance of the central region. The envelope mismatch is the major cause of emittance growth and halo formation. Thus it is important to study the evolution of space charge dominated beam to avoid beam loss during acceleration.

In this paper we have studied the behavior of space charge dominated beam envelopes in a compact cyclotron along the accelerated orbits using the coupled beam envelope equations assuming a uniform ellipsoidal beam bunch. The input beam parameters have been optimized to minimize the amplitude growth and oscillations. We have also estimated the maximum beam current that can be transported in the specified aperture of the compact cyclotron.

CENTRAL ORBITS AND ENVELOPE EQUATIONS

The magnet of 10 MeV cyclotron consists of four sectors with maximum hill field equal to 1.5 T. The hill and valley gaps are equal to 5 cm and 50 cm respectively [2]. A deep valley structure has been used to get vertical betatron tune $\upsilon_y > 0.5$ at all radii for handling the space charge defocusing force. Apart from using a high dee voltage

(V_g = 125 kV) and high injection energy (80 keV), we have chosen a low average magnetic field (0.689 T) and hence a large extraction radius (~ 65 cm) for 10 MeV cyclotron to have a reasonable turn separation at the extraction radius. A 3D code MagNet was used to calculate the magnetic field in the median plane. The values of the magnetic betatron tunes were estimated from the equilibrium orbit code GENSPEO [3]. The electric vertical betatron tune was estimated using the first order theory [4, 5]. The variation of the radial υ_x and vertical υ_y betatron tunes as a function of orbit radius R is shown in Fig. 1(a). The vertical betatron tune υ_y is the resultant of magnetic and electric tunes shown by dashed line. We have used values of these υ_x and υ_y in the beam envelope calculations. The coordinates and velocity of the central ion trajectory obtained at the inflector exit was used as input for tracing the central orbits in the cyclotron (Fig. 1(b)). The electric field in the median plane at four gaps was approximated by a Gaussian function.

For an ellipsoidal bunch moving along the accelerated orbit in the cyclotron focusing channel, the differential equations for beam envelopes X(s) and Y(s) in the two transverse planes are given by [6],

$$X'' + \frac{(\beta\gamma)'}{(\beta\gamma)}X' + \left[\frac{\upsilon_x^2}{R^2} - \frac{3Ic}{2I_0\beta^2\gamma^3 f_{rf}}\frac{1}{X^3}G\left(\frac{Y}{X},\frac{Z}{X}\right)\right]X \frac{\varepsilon_{nx}^2}{\beta^2\gamma^2X^3} = 0 \qquad (1a)$$

$$Y'' + \frac{(\beta\gamma)'}{(\beta\gamma)}Y' + \left[\frac{\upsilon_y^2}{R^2} - \frac{3Ic}{2I_0\beta^2\gamma^3 f_{rf}}\frac{1}{Y^3}G\left(\frac{X}{Y},\frac{Z}{Y}\right)\right]Y - \frac{\varepsilon_{ny}^2}{\beta^2\gamma^2Y^3} = 0 \qquad (1b)$$

The average beam current $I = (4/3)\pi XYZn_b q f_{rf} = Q_b f_{rf}$. Here Q_b is the total charge in the bunch which remains constant during the motion, f_{rf} is the rf frequency, ε_{nx} and ε_{ny} are the normalized emittances in the x and y planes respectively and I_0 = 31 MA for protons. The expression for G is given in detail in ref. [7] in terms of the elliptic integrals. For space charge dominated beam with acceleration it is not possible to obtain the matched beam sizes analytically. In such situation one needs to solve Eqs. (1) along the accelerated orbits and then to optimize the initial conditions for which the envelope functions X(s) and Y(s) exhibit minimum amplitude of oscillations.

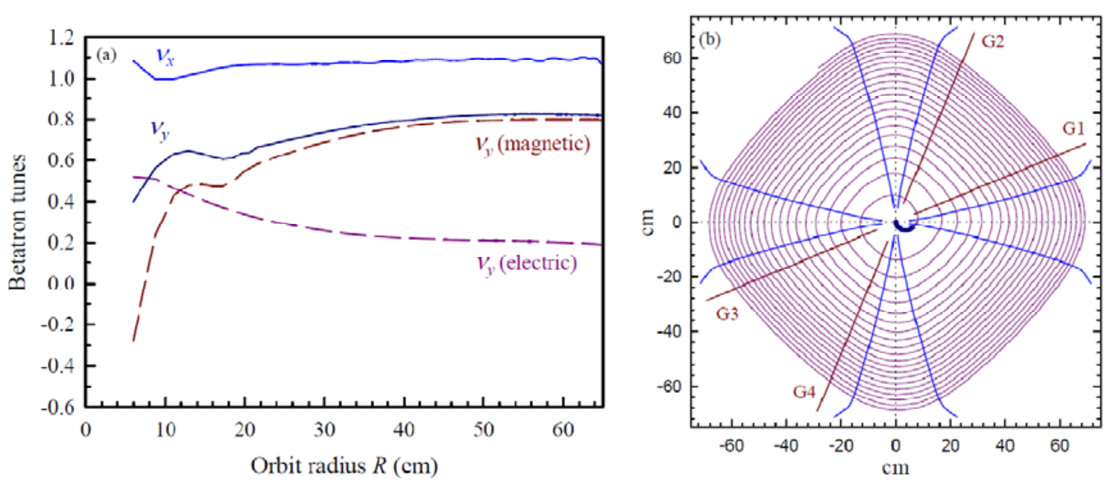

FIGURE 1. (a) Betatron tunes as a function of orbit radius R. (b) Position of the inflector, location of the accelerating gaps (V_g = 125 kV) and accelerated orbits of proton from 80 keV to 10 MeV.

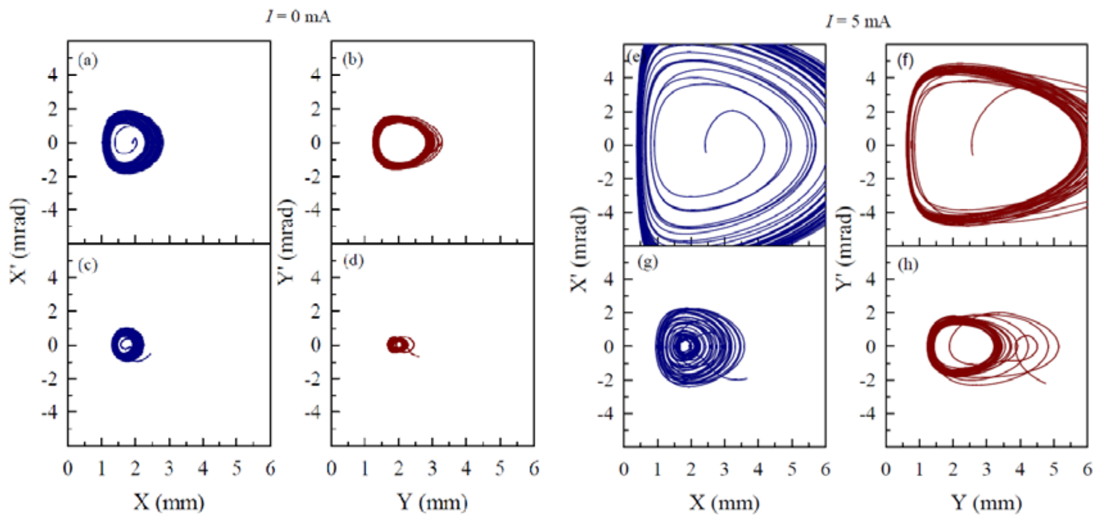

FIGURE 2. Plot of envelope slop with size up to final radius for $I = 0$ mA and 5 mA for both planes. Plots 2(a) and 2(b) were obtained with matched initial beam sizes $X_m = 1.92$ mm and $Y_m = 3.26$ mm at the injection. Plots shown in 2(c) and 2(d) were obtained after optimization. Plots 2(e) and 2(f) are for $I = 5$ mA with same input conditions as in 2(c) and 2(d). Optimized plots are shown in 2(g) and 2(h).

RESULTS AND DISCUSSION

For the numerical simulation we have used the normalized emittances in both the planes equal to 0.7 πmmmrad. The initial bunch size corresponding to 30^0 of rf at the injection energy of 80 keV and rf frequency of 42 MHz, is equal to 8 mm. The values of $(\beta\gamma)'/(\beta\gamma)$ and bunch size along path length s have been estimated from the change of energy of the particle using the data obtained from the orbit integration code. Since there is a wide variation of betatron tunes with radius (see Fig. 1(a)), the matched beam sizes at different orbit radii for different values of the beam current are different.

The size and slope of the beam envelopes as a function of path length s along the accelerated orbit for different conditions were obtained by solving Eqs. (1). Figure 2(a) and 2(b) are the results for $I = 0$ mA and with acceleration ($V_g = 125$ kV). The initial beam sizes are the matched beam sizes $X_m = 1.92$ mm and $Y_m = 3.26$ mm at the injection radius (7.05 cm) and with the phase ellipses as shown in Fig. 3(a). We see that due to the acceleration even at very low current there is a considerable growth (~ 3.2 mm) in the amplitude of the envelope in both planes. Therefore these initial conditions are not at all suitable. Figure 2(c) and 2(d) shows the results after optimizing the initial phase ellipses (Fig. 3(b)) in both planes.

The behavior of beam envelopes for 5 mA is shown in Fig. 2(e) to Fig. 2(h). The input conditions of the beam in Fig. 2(e) and 2(f) are the same as the optimized input conditions for $I = 0$ mA, $V_g = 125$ kV (Fig. 3(b)). As we see these input conditions produce more amplitude growth in the beam envelopes. The behavior of optimized beam envelopes and slopes is shown in Fig. 2(f) and 2(g) and the corresponding initial conditions are shown in Fig. 3(c). We have also estimated the maximum limiting current that can be transpoted through the cyclotron within 6 mm half aperture. Figure 3(d) shows beam envelopes up to 21 turns with optimized initial conditions. The limiting current is approximately 7 mA. The maximum height of the dee from the median plane is equal to 15 mm. Therefore, a 5 mA beam current can be easily controlled in the present design of the cyclotron. A scaling of υ_x either up or down from the present value reduces the amplitude and oscillations considerably. Fig 3(e) shows the envelopes where υ_x and υ_y are scaled up by 1.15 times. In a compact cyclotron, one can manipulate the values υ_y by changing the flutter and shape of the sectors. It is not possible to change the profile of υ_x as desired in an isochronous cyclotron because it follows the profile of relativistic term γ. So the best way to control the beam envelope growth is then to optimize the initial beam conditions.

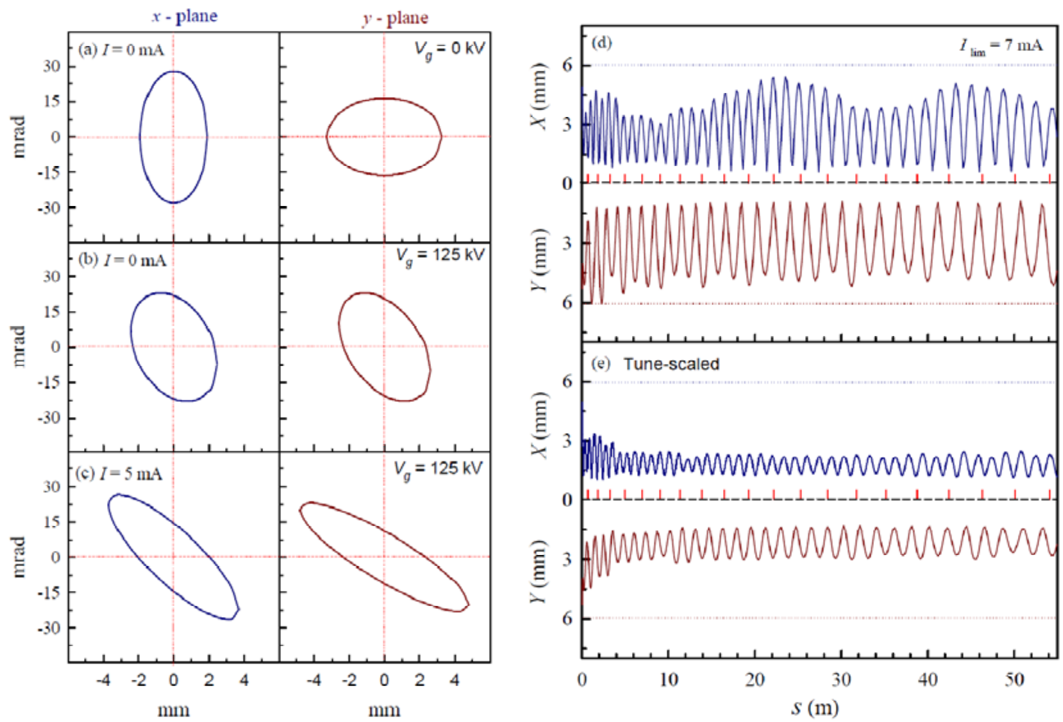

FIGURE 3. Input ellipses for (a) matched envelope sizes for $I = 0$ mA, (b) optimized envelopes with acceleration and $I = 0$ mA, (c) optimized envelopes with acceleration and $I = 5$ mA. (d) beam envelopes up to 10 MeV with initial conditions $X_0 = 4.9$ mm, $X'_0 = -30$ mrad, $Y_0 = 5.3$ mm, $Y'_0 = -35$ mrad and (e) envelopes when betatron tunes are scaled by 1.15 times with same initial conditions as in (d).

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