Bi-stability in Accelerator Driven U^{233} Breeders

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Abstract. We present a model to understand breeding in Accelerator Driven Systems (ADSs). We show that even with non-fissioning, non-power-producing targets such as *Pb*, it is possible to choose the fuel irradiation time so that the breeder produces sufficient power to drive the accelerator and export the balance to the grid, without significantly diminishing the U^{233} breeding rate.

Using the non-linear equations of our model, we show that under certain circumstances, the equilibrium discharge fluence (and power, rate of breeding, *etc.*) is a triple valued function of the irradiation time. For the solution corresponding to the middle value, the fluence decreases with increasing irradiation time and the equilibrium appears to be unstable. For the two solutions with values below and above middle value, the fluence increases with increasing irradiation time and the equilibrium appears to be stable. Thus the system looks like a typical bi-stable one.

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INTRODUCTION

Research on ADS is being carried out around the world primarily with the objective of waste transmutation. Due to the large amounts of *Pu* and enriched *U* available from the civil and military programs, there is little incentive for breeding. Therefore, the present focus has been shifted from power generation using Th fuels (which was the initial objective of Carlo Rubia's proposal [1]) to waste transmutation using ADS However, in India there is a strong incentive for breeding and far little incentive for waste transmutation immediately though this may change with expected large scale expansion nuclear power program. Moreover due to large Th deposits there is a clear incentive to develop Th related technologies. Th has the additional advantage that it produces very little transuranic waste.

In this article, we will present our study on ADS breeder for both the thermal and the fast spectrum. We found that even with non-fissioning, non-power-producing targets such as *Pb* and *LBE* it is possible to choose the fuel irradiation time so that the breeder produces sufficient power to drive the accelerator and some more power for the grid, without significantly diminishing the U^{233} breeding rate. More interestingly, we found that under certain circumstances, the equilibrium discharge fluence, power and rate of breeding are triple valued function of the irradiation time. For the solution corresponding to the middle value, the fluence decreases with increasing irradiation time and the equilibrium appears to be unstable. For the two solutions with values below and above middle value, the fluence increases with increasing irradiation time and the equilibrium appears to be stable. Thus the system resembles a typical bi-stable system.

MODEL FOR THE ADS BREEDER

To get a quantitative understanding of the breeding process in *ADSs*, we set up the following simple model where Th bundles are introduced into the sub-critical core and removed at a fixed rate. The core composition is kept constant by shuffling the bundles around. We assume that whatever neutrons leak out of the main power producing region of the core are absorbed in a Th blanket having low presence of U^{233}. The leakage from such a core can be made practically zero by moving bundles from the blanket in to the core and discharging the irradiated bundles from the core. Thus we can have an equilibrium core. The problem is to find k_{eff}, power, flux, average U^{233} fraction and the rate of U^{233} production of such an equilibrium core as a function of irradiation time (t_d). For simplicity, we shall explicitly consider only two isotopes, namely Th and U^{233}. The fission products are lumped together and treated with an effective absorption cross-section.

The evolution equation for the Th and U^{233} can be written as:

$$dC_{Th}/du = -\sigma_T^{Th} C_{Th} \quad (1)$$

$$dC_U/du = \alpha \sigma_T^{Th} C_{Th} - \sigma_T^U C_U. \quad (2)$$

Where, C represents the concentration and σs are the microscopic reaction cross-sections, u is the fluence (φt) and $\alpha = \lambda_{Pa}/(\lambda_{Pa} + \sigma_c^{Pa}\varphi)$. The average concentrations (represented with bar) at the discharge fluence (u_d) can be found by solving the above equations and using $\overline{C(u_d)} = 1/u_d \int_0^{u_d} C(u)du$. Finally, the k_{eff} and flux of such system are given by the following non-linear equations:

$$k_{eff} = \nu\sigma_f^U \overline{C_U(u_d)} / \left(\sigma_T^{Th}\overline{C_{Th}(u_d)} + \sigma_T^U \overline{C_U(u_d)} + \Sigma_{l0} + \overline{\Sigma_l(u_d)}\right) \quad (3)$$

$$\varphi = S / \left(\sigma_T^{Th}\overline{C_{Th}(u_d)} + (\sigma_T^U - \nu\sigma_f^U)\overline{C_U(u_d)} + \Sigma_{l0} + \overline{\Sigma_l(u_d)}\right). \quad (4)$$

Where, Σ_{l0} is the losses due to parasitic capture (other than capture in fission products). The average equilibrium capture in fission products, $\overline{\Sigma_l(u_d)}$, has to be obtained from a burnup core (*e.g.* McBurn [2]). One can convert from discharge fluence (u_d) to the discharge time (t_d) in days using the relation:

$$t_d = \frac{u_d}{\varphi} = \frac{u_d}{S}\left(\sigma_T^{Th}\overline{C_{Th}(u_d)} + (\sigma_T^U - \nu\sigma_f^U)\overline{C_U(u_d)} + \Sigma_{l0} + \overline{\Sigma_l(u_d)}\right). \quad (5)$$

Unfortunately, t_d is not always a monotonically increasing function of u_d and in that case the interpretation becomes difficult. However in most practical situations, monotonicity holds and we can use the above set of equations to unambiguously calculate all quantities of interest. We assume this to be the case and the non-monotonic case will be discussed separately.

RESULTS

The results of calculations using the above model are illustrated in Fig. 1 showing the variation of different parameters with exposure time at discharge for heavy water

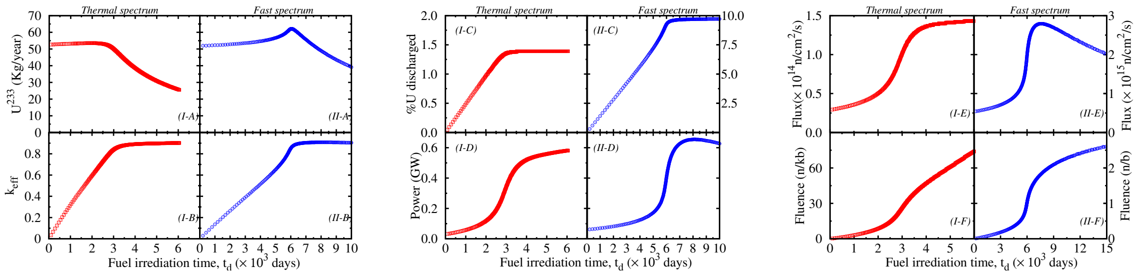

FIGURE 1. Results are shown for thermal ($I-A:F$) and fast ($II-A:F$) spectrum. We see that the rate of U^{233} production for the thermal system there is a slow and small rise till a peak is reached and the rate falls off thereafter. k_{eff} increases almost linearly and saturates later on at about 0.9. Similarly the U^{233} fraction in the fuel first increases linearly and rapidly flattens off at a value of about 1.4%. The power and the flux follow a S type curve and unlike the k_{eff} and the fissile fraction the saturation is much slower. Fluence is monotonically increasing with t_d The fast system shows a similar behaviour but with some important differences. Due to the much greater equilibrium fissile fraction in fast systems, the discharged fuel has a much higher fraction of U^{233} than in a thermal breeder. Moreover due to the higher value of $\eta\varepsilon$ and much lower parasitic capture in fission products, the fast system has a somewhat higher peak breeding rate, a higher k_{eff} and correspondingly higher power.

moderated and fast Th blankets driven by a 1 *GeV* 30 *mA* proton beam on *Pb*. The various input data used are spectrum averaged values based on lattice calculations. The fixed parasitic absorption is assumed to be 8% and 10% of the absorption in Th for the thermal and fast systems respectively. Finally the source strength is assumed to be 25 neutrons per 1 *GeV* proton. It is clear from the figures that if the fuel is discharged at a lower burnup, the k_{eff} and the energy gain are lower and hence a larger fraction of the produced power goes to the accelerator. The breeding rate is however higher and the system becomes an accelerator breeder producing its own power or even a net exporter of power. At still lower exposures, the system cannot produce the power required to drive the accelerator. However, irradiating the Th sufficiently long, it is possible to reach close to the maximum fissile fraction, produce enough power to run the accelerator and possibly some extra power for the grid without significant loss of the fissile U^{233} production rate. Irradiating the fuel longer than that can result in greater power production but causes a significant reduction in the rate of production of U^{233}. Moreover the exposure of the fuel goes well beyond the limits of experience with water cooled reactors.

The non-monotonic case: bi-stability

We had mentioned the possibility that t_d might not always be a monotonically increasing function of u_d. To see this we differentiate Eq. 5 *w.r.t.* u_d. Since the right hand side is a product of the discharge exposure and the mean of various quantities (essentially representing the net absorption cross section) it is clear that the derivative is simply the net absorption cross section at discharge. Clearly when $k_\infty = 1$, this quantity is zero and we have a local extremum of t_d. On the other hand when $k_\infty > 1$, the derivative becomes negative and the exposure starts increasing with decreasing discharge time. This may

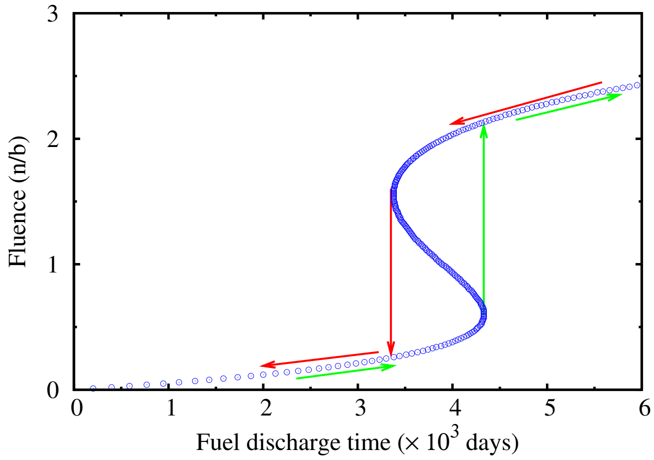

FIGURE 2. Showing variation of fluence with discharge time of the fuel. The path taken by the system in the case of increasing (decreasing) discharge time is shown by green (red) arrows.

seem counter-intuitive but can be understood as being due to a more rapid increase of flux in this regime. In fast systems it can happen that as the exposure is increased, the k_∞ of the fuel increases beyond unity. Ultimately it has to start decreasing due to accumulation of fission products and comes back below unity. The second time it crosses unity we have a minimum. Thus t_d as a function of u_d first goes through a local maximum and then through a local minimum before increasing again.

Since all quantities depicted above are explicit functions of u_d, we should know the value of u_d for any stipulated value of t_d. Due to the above behaviour, in the region between the two extrema, the solution of Eq. 5 for u_d has three real roots. The root between the two extrema is probably unstable (see Fig. 2), and the other two are stable, *i.e.* it appears to be a bistable system in this region. Which of the two stable states the system goes into will be decided by the history.

CONCLUSIONS

Our results indicate that depending on the exposure at discharge, a whole range of ADS U^{233} breeder systems is possible - from breeders importing their power requirements to those that are self sufficient to those that are net exporters of power and finally the Th burners. We also showed that under certain circumstances, the equilibrium discharge fluence becomes a multivalued variable and there by indicating the possibility of a bistable system.

REFERENCES

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